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Analysis of Entropy Generation between porous disks due to Micropolar Fluid flow with MHD effect

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--***ABSTRACT***--
This paper examines the magneto hydrodynamic flow of an electrically conducting micropolar fluid flow between parallel porous disks with constant uniform suction through the surface of the disks. The fluid is subjected to an external transverse magnetic field. The governing equations of the fluid flow are linearized using quasilinearization method and further, solved by the Chebyshev spectral collocation method. The numerical data for velocity, microrotation and temperature fields are used to evaluate entropy generation and Bejan number. It has been found that the entropy generation decreases with increase in Hartman number. Heat transfer irreversibility dominates at the lower and upper disks whereas fluid friction irreversibility dominates at the center of the parallel disks are observed from all Bejan number profiles.

KEYWORDS;- *Parallel disks, Micropolar fluid, Magnetic field, Entropy, Bejan number*

I. INTRODUCTION

In most of the thermal systems, the thermal efficiency can be defined as a ratio of actual efficiency of thermal system to reversible thermal efficiency where the applied conditions are same. The fluid flow and heat transfer processes are intrinsically irreversible, which leads to increase entropy generation and useful energy destruction. Taking this into consideration, worldwide research is going on to reduce the entropy generation. Bejan [1] was the pioneer work on entropy generation. He first presented the second law aspect of heat transfer using different examples of fundamental forced convection problem. Bejan [2, 3] investigated entropy generation minimization and showed the fundamental importance of entropy minimization for efficient engineering processes. Ever since numerous researches have been conducted on entropy generation for different geometric configurations.

The fluid flow between parallel disks is a topic of much interest to mathematicians and engineers. This type of flow finds several applications such as hydrodynamical machines and apparatus, disk type heat and mass exchangers, centrifugal manometers, viscometry, cooling of gas turbine disks, and rotor-stator systems. Considerable research studies were carried out to investigate the fluid flow and entropy generation between the parallel disks. Rashidi et al. [4] discussed the entropy generation under the effects of magnetic interaction number and slip factor with variable properties over rotating porous disk. Feng et al. [5] considered the problem of entropy generation minimization for asymmetric vascular networks in a disc shaped body. Rashidi et al. [6] performed the analysis of entropy generation over the rotating porous disk with variable physical properties.

Most of the studies on entropy generation are related to Newtonian fluids. But the fluids used in Engineering and industrial processes, such as poly liquid foams and geological materials, exhibit flow properties that cannot be explained by Newtonian fluid flow model. Several non-Newtonian fluid flow models have been proposed to explain the behavior of such fluids. Among these, micropolar fluids introduced by Eringen [7] have distinct features, such as the local structure effects which are microscopic and micro motion of elements of the fluid, the presence of stresses due to couple, body couples and non-symmetric stress tensor. More interesting aspects of the theory and application of micropolar fluids can be found in the books of Lukaszewicz [8]. The study of flow through parallel disks related to micropolar fluids has received a considerable interest. Takhar et al. [9] considered the problem of micropolar fluid flow through an enclosed rotating disc with suction or injection. Ashraf and Wehgal [10] studied the flow and heat transfer of a steady incompressible viscous electrically conducting micropolar fluid between two stationary infinite parallel porous disks in the presence of a uniform magnetic field.

To the best of author's knowledge, the entropy generation analysis due to micropolar fluid flow between porous discs has not been the subject of previous studies yet. Thus, we have considered the problem of entropy generation of MHD flow in porous discs due to micropolar fluid flow. The effects of coupling number, Hartman number, Reynolds number and Brinkman number on entropy generation and Bejan number are examined. The obtained results can be applied for the design of thermal systems with the less irreversibility sources.

II. MATHEMATICAL FORMULATION

Consider a steady, laminar, incompressible, axisymmetric flow of an electrically conduct- ing micropolar fluid flow between two parallel porous disks of infinite radii positioned at the planes $z = \pm h$. A constant magnetic field of strength B_0 is imposed perpendicular to the plane of disks. The magnetic Reynolds number is very small so that induced magnetic field can be neglected in comparison to applied magnetic field. Let the velocity and microrotation vectors are chosen as $\left(u = u(r,z), 0, w = w(r,z)\right)$, and $\left(0, \sigma(r,z), 0\right)$ where *u* and *w* are radial and axial velocity components and *σ* is the transverse microrotation component. The temperature is taken as *T (r, z)*.

The governing equations for MHD incompressible micropolar fluid are given by

$$\frac{\partial u}{\partial r} = \frac{u}{r} + \frac{1}{h}\frac{\partial u}{\partial \eta} \tag{1}$$

$$\rho\left[u\frac{\partial u}{\partial r} + \frac{w}{h}\frac{\partial u}{\partial \eta}\right] = -\frac{\partial p}{\partial r} + (\mu+\kappa)\left[\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{1}{h^2}\frac{\partial^2 u}{\partial \eta^2}\right] - \frac{\kappa}{h}\frac{\partial \sigma}{\partial \eta} - \sigma_e u B_0^2 \tag{2}$$

$$\rho\left[u\frac{\partial w}{\partial r} + \frac{w}{h}\frac{\partial w}{\partial \eta}\right] = -\frac{1}{h}\frac{\partial p}{\partial \eta} + (\mu+\kappa)\left[\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{h^2}\frac{\partial^2 w}{\partial \eta^2}\right] + \kappa\left(\frac{\partial \sigma}{\partial r} + \frac{\sigma}{r}\right) \tag{3}$$

$$\rho j^*\left[u\frac{\partial \sigma}{\partial r} + \frac{w}{h}\frac{\partial \sigma}{\partial \eta}\right] = \kappa\left(\frac{1}{h}\frac{\partial u}{\partial \eta} - \frac{\partial w}{\partial r}\right) - 2\kappa\sigma + \gamma\left[\frac{\partial^2 \sigma}{\partial r^2} + \frac{1}{r}\frac{\partial \sigma}{\partial r} - \frac{\sigma}{r^2} + \frac{1}{h^2}\frac{\partial^2 \sigma}{\partial \eta^2}\right] \tag{4}$$

$$\begin{aligned}\rho C_p\left[u\frac{\partial T}{\partial r} + \frac{w}{h}\frac{\partial T}{\partial \eta}\right] &= K_f\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{h^2}\frac{\partial^2 T}{\partial \eta^2}\right] + \\ &(2\mu+\kappa)\left[\left(\frac{\partial u}{\partial r}\right)^2 + \frac{u^2}{r^2} + \frac{1}{h^2}\left(\frac{\partial w}{\partial \eta}\right)^2 + \frac{1}{2}\left(\frac{1}{h}\frac{\partial u}{\partial \eta} + \frac{\partial w}{\partial r}\right)^2\right] + \frac{\kappa}{2}\left[\frac{1}{h}\frac{\partial u}{\partial \eta} - \frac{\partial w}{\partial r} - 2\sigma\right]^2 + \\ &\gamma\left[\left(\frac{\partial \sigma}{\partial r}\right)^2 + \frac{\sigma^2}{r^2} + \frac{1}{h^2}\left(\frac{\partial \sigma}{\partial \eta}\right)^2\right] - 2\beta\frac{\sigma}{r}\frac{\partial \sigma}{\partial r}\end{aligned} \tag{5}$$

where $\eta = \dfrac{z}{h}$

The boundary conditions are given by

$$u(r,\eta) = 0, \quad w(r,\eta) = \mp V_0, \quad \sigma(r,\eta) = 0 \quad \text{at} \quad \eta = \pm 1 \tag{6a}$$

$$T'(r,\eta) = T_1 \quad at \quad \eta = -1 \quad T(r,\eta) = T_2 \quad \text{at} \quad \eta = 1 \tag{6b}$$

Introducing the following transformation

$$u = \frac{V_0 r}{2h} f'(\eta), \quad w = -V_0 f(\eta), \quad \sigma = \frac{V_0 r}{2h^2} g(\eta) \tag{7}$$

in equations (2), (3) and (4), we get the following equations in the dimensionless form:

$$\frac{1}{1-N} f^{iv} + Reff''' - \frac{N}{1-N} g'' - Ha^2 f'' = 0 \tag{8}$$

$$\frac{N}{1-N}\left(f'' - 2g\right) - Rea_j\left(\frac{1}{2} f'g - fg'\right) + \frac{N(2-N)}{m^2(1-N)} g'' = 0 . \tag{9}$$

Where $N = \dfrac{\kappa}{\kappa+\mu}$ is the coupling number, $\mathrm{Re} = \dfrac{\rho V_0 h}{\mu}$ is the Reynolds number based on the injection or suction velocity $Ha = B_0 r_1 \sqrt{\dfrac{\sigma_e}{\mu}}$ is the Hartman number, $m^2 = \dfrac{h^2\kappa(2\mu+\kappa)}{\gamma(\mu+\kappa)}$ is the micropolar parameter, $a_j = \dfrac{j}{h^2}$ is the micro inertia parameter. Equation (5) together with (7), suggests that the form of temperature may be taken as

$$T = T_1 + \frac{\mu V_0}{\rho h^3 C_{\mathrm{P}}}\left[\theta_1(\eta) + \frac{r^2}{h^2}\theta_2(\eta)\right] \tag{10}$$

Substituting (10) in (5) and equating the coefficients of r2 and the terms without r on both sides of the equation thus obtained, we get

$$\theta_1'' + 4\theta_2 + \Pr\mathrm{Re}\, f\theta_1' + \frac{\Pr Re}{2}\left[3\left(\frac{2-N}{1-N}\right)f'^2 + \left(\frac{N(2-N)}{m^2(1-N)} - B\right)g^2\right] = 0 \tag{11}$$

$$\theta_2'' + \frac{\Pr\mathrm{Re}}{8}\left[\left(\frac{2-N}{1-N}\right)f''^2 + \frac{N}{1-N}\left(f'' - 2g\right)^2\right] + \Pr\mathrm{Re}\left[\frac{1}{4}\frac{N(2-N)}{m^2(1-N)}g'^2 + f\theta_2' - f'\theta_2\right] = 0 \tag{12}$$

Where $\Pr = \dfrac{\mu C_p}{K_f}$ is the Prandtl number. The dimensionless form of temperature can be written as

$$\theta = \frac{T - T_1}{T_2 - T_1} = Ec\left[\theta_1 + \hat{x}^2\theta_2\right] \tag{13}$$

Where $Ec = \dfrac{\mu V_0}{\rho\left(T_2 - T_1\right)hC_p}$ is the Eckert number and $\hat{x} = \dfrac{r}{h}$ is the dimensionless radial variable. The boundary conditions (6) in terms of f, g, θ_1 and θ_2 are:

$$f(-1) = -1,\quad f'(-1) = 0,\quad g(-1) = 0,\quad \theta_1(-1) = 0,\quad \theta_2(-1) = 0 \tag{14a}$$

$$f(1) = 1,\quad f'(1) = 0,\quad g(1) = 0,\quad \theta_1(1) = \frac{1}{Ec},\quad \theta_2(1) = 0 \tag{14b}$$

III. METHOD OF SOLUTION

The system of Eqns. (8), (9),(11),(12) are linearized using the quasilinearization method. This quasilinearization method (QLM) is a generalization of the Newton-Raphson method and was proposed by Bellman and Kalaba [11] for solving nonlinear boundary value problems. In this method the iteration scheme is obtained by linearizing the nonlinear component of a differential equation using the Taylor series expansion.

Let the $f_r, g_r, \theta_{1,r}$ and $\theta_{2,r}$ be an approximate current solution and $f_{r+1}, g_{r+1}, \theta_{1,r+1}$ and $\theta_{2,r+1}$ be an improved solution of the system of equations (8), (9),(11) and (12). By taking Taylor series expansion of non-linear terms in (8), (9),(11) and (12) around the current solution and neglecting the second and higher order derivative terms, we get the linearized equations as:

$$\frac{1}{1-N}f_{r+1}^{iv} + a_{1,r}f_{r+1}''' - Ha^2 f_{r+1}'' + a_{2,r}f_{r+1} - \frac{N}{1-N}g_{r+1}'' = q_{1,r} \tag{15}$$

$$\frac{N}{1-N}f_{r+1}'' - b_{1,r}f_{r+1}' + b_{2,r}f_{r+1} + \frac{N(2-N)}{m^2(1-N)}g_{r+1}'' + b_{3,r}g_{r+1}' - b_{4,r}g_{r+1} = q_{2,r} \tag{16}$$

$$c_{1,r}f_{r+1}' + c_{2,r}f_{r+1} + c_{3,r}g_{r+1} + \theta_{1,r+1}'' + c_{4,r}\theta_{1,r+1}' + 4\theta_{2,r+1} = q_{3,r} \tag{17}$$

$$d_{1,r} f''_{r+1} - d_{2,r} f'_{r+1} + d_{3,r} f_{r+1} + d_{4,r} g'_{r+1} + d_{5,r} g_{r+1} + \theta''_{2,r+1} + d_{6,r} \theta'_{2,r+1} - d_{7,r} \theta_{2,r+1} = q_{4,r} \tag{18}$$

Where the coefficients $a_{s,r}, b_{s,r}, c_{s,r}, d_{s,r}, s = 1, 2, ...$ are known functions in terms of $f_r, g_r, \theta_{1,r}$ and $\theta_{2,r}$ and their derivatives.

The above linearized equations (15)-(18) are solved using the Chebyshev spectral collocation method [12]. The functions $f_{r+1}, g_{r+1}, \theta_{1,r+1}$ and $\theta_{2,r+1}$ and their derivatives are approximated in terms of Chebyshev polynomials $T_k(\xi) = \cos[\kappa \cos^{-1} \xi]$ at Gauss-Lobatto collocation points $\xi_j = \cos\frac{\pi j}{J}, j = 0, 1, 2, ..., J, J$ is the number of collocation points, leads to a $(4J+4)\times(4J+4)$ matrix system. Incorporating boundary conditions and solving the resulting matrix system, we get the solution.

IV. ENTROPY GENERATION

According to Bejan [3], the dimensionless entropy generation number N_s is the ratio of the volumetric entropy generation rate to the characteristic entropy generation rate. Thus the entropy generation number is given by

$$N_s = Ec^2\left(4\hat{x}^2\theta_2^2 + \left(\theta_1' + \hat{x}^2\theta_2'\right)^2\right) + \frac{Br}{T_p}\left\{\left(\frac{2-N}{1-N}\right)\left(\frac{3}{2}f'^2 + \frac{1}{8}\hat{x}^2 f''^2\right) + \frac{1}{2}\frac{N}{1-N}\hat{x}^2\left(\frac{1}{2}f'' - g\right)^2\right.$$
$$\left. + \frac{N(2-N)}{m^2(1-N)}\left(\frac{1}{2}g^2 + \frac{1}{4}\hat{x}^2 g'^2\right) - \frac{B}{2}g^2\right\} + \frac{Ha^2}{4}\frac{Br}{T_p}\hat{x}^2 f'^2 \tag{19}$$

Where $T_p = \frac{T_2 - T_1}{T_1}$ is the dimensionless temperature difference, and the characteristic entropy generation rate is $\frac{\kappa_f (T_2 - T_1)^2}{h^2 T_1^2}$. The equation (19) can be expressed alternatively as follows

$$N_s = N_h + N_v + N_m \tag{20}$$

The first term on the right hand side of this equation denotes the entropy generation due to heat transfer irreversibility, the second term represents the entropy generation due to viscous dissipation and third term represents due to magnetic effect.

To evaluate the irreversibility distribution, the parameter Be(Bejan number), which is the ratio of entropy generation due to heat transfer to the overall entropy generation (20) is defined as follows

$$Be = \frac{N_h}{N_h + N_v + N_m} \tag{21}$$

V. RESULTS AND DISCUSSION

The entropy generation for micropolar fluid flow between parallel disks is studied in this paper. Entropy generation in the flow field due to heat transfer, fluid friction and magnetic field is formulated. The influence of various parameters on velocity, microrotation, temperature, entropy generation and Bejan number are examined. To study the effects of N, Ha, Re, Ec and Br computations were carried out by taking $B = 1.5, \hat{x} = 0.5, m = 1, \Pr = 1, a_j = 0.001, T_p = 1.$

Fig. 1 presents the effect of coupling number (N) on the entropy generation and Bejan number. The coupling number N characterizes the coupling of linear and rotational motion arising from the fluid particles. In the case of *N = 0* (i.e. as *κ* tends to zero) the micropolarity is absent and fluid becomes nonpolar fluid. With a large value of N , the effect of microstructure becomes significant, whereas with a diminished value of N the individuality of the substructure is much less articulated. As N increases the entropy generation and Bejan number are also increasing as shown in Figs. 1(a) and Fig. 1(b). From the Bejan number profile clearly it is observed that the heat transfer irreversibility dominates near the disks and fluid friction irreversibility dominates at the center of the parallel disks.

Fig. 2 shows the effect of MHD effect on the entropy generation and Bejan number. As seen from Fig. 2(a) an increase in the Hartmann number leads to increase in the entropy generation number in the entire region. In this case, although the contribution of the magnetic field to entropy generation is increased a little, the magnetic field reduces the contribution of fluid friction to entropy generation with the lowering of the shear stresses and hence the velocity gradients increase between the disks. It is noticed from Fig. 2(b) that the Bejan number decreases with increase in Hartman number.

It is observed from Fig. 3(a) that the entropy generation increases with increase in suction Reynolds number. Also it is interesting to note that the less entropy generation is observed at the center of the parallel disks due to less contribution of velocity and temperature gradients. Fig. 3(b) depicts that the Bejan number increases with increase in Reynolds number.

The influence of Eckert number on the entropy generation and Bejan number are presented in Fig. 4. It is observed from Fig. 4(a) that the entropy generation increases with increase in Ec. Fig. 4(b) describes that the Bejan number increases as increase in Eckert number values.

It is observed from Fig. 5(a) that the entropy generation increases with increase in Brinkman number. Fig. 5(b) shows the distribution of Bejan number with variation of Brinkman number. As *Br* increases, Bejan number attains two maximum values, one at the lower disk and the other at the upper disk. This is due to the occurrence of maximum temperature gradients near the two disks. With increasing axial distance, *Be* falls to zero at the center of the disks and then rises for variation of all parameters.

VI. CONCLUSIONS

In this paper, micropolar fluid flow between parallel disks with constant temperatures at the disks under the MHD effect has been studied. The numerical values are obtained for velocity, microrotation and temperature by applying Chebyshev spectral collocation method. The entropy generation is found at every point η between the parallel disks is found. The effects of coupling number, Hartman number, Reynolds number, Eckert number and Brinkman number on entropy generation and Bejan number is studied through graphs.

1. As magnetic field strength increases, entropy generation and Bejan number decreases between the parallel disks.
2. An increase in Eckert number results in an increase in the entropy generation and Bejan number.
3. Heat transfer irreversibility dominates at the lower and upper disks whereas fluid friction irreversibility dominates at the center of the parallel disks

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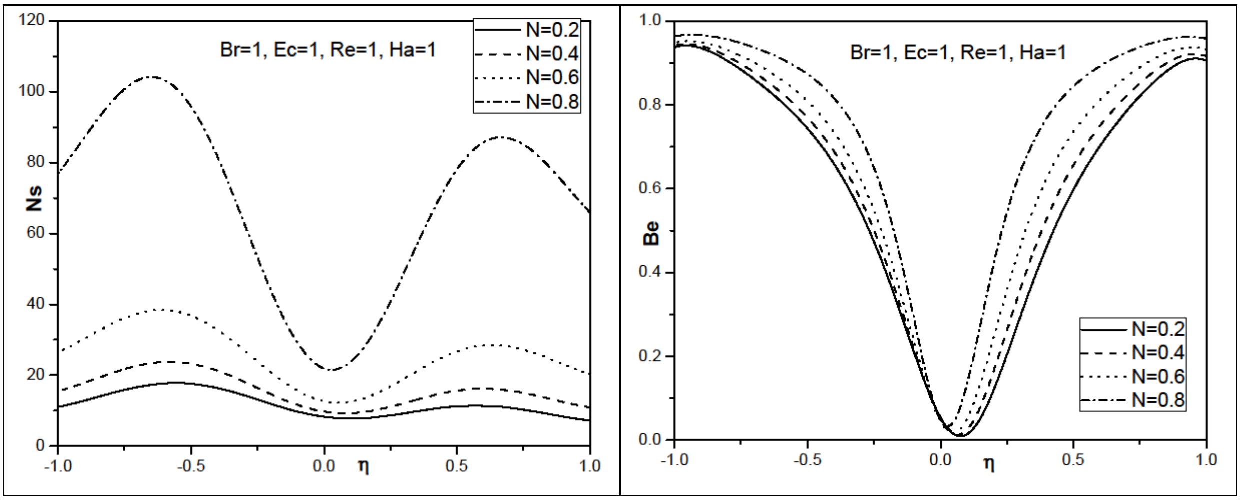

Figure 1: Effect of coupling number on the entropy generation and Bejan number

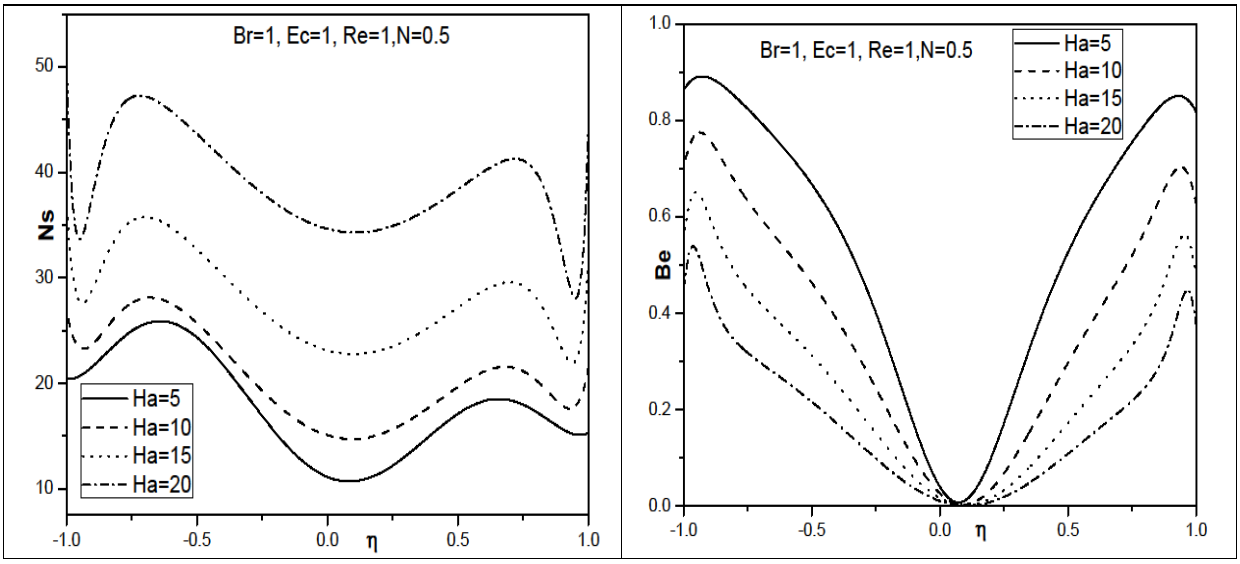

Figure 2: Effect of Hartman number on the entropy generation and Bejan number

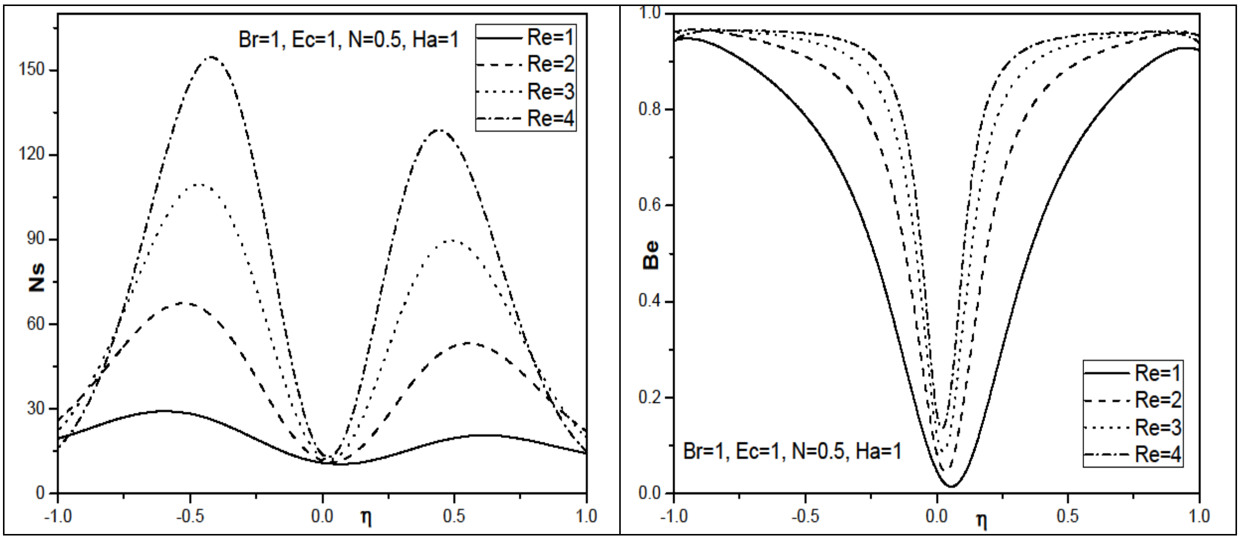

Figure 3: Effect of Reynolds number on the entropy generation and Bejan number

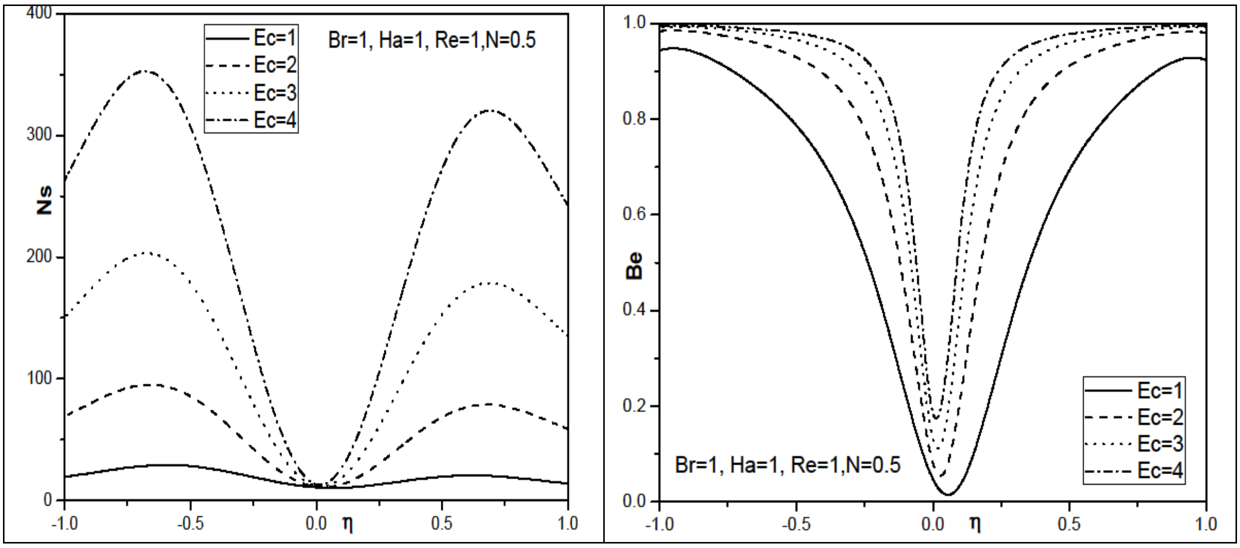

Figure 4: Effect of Eckert number on the entropy generation and Bejan number

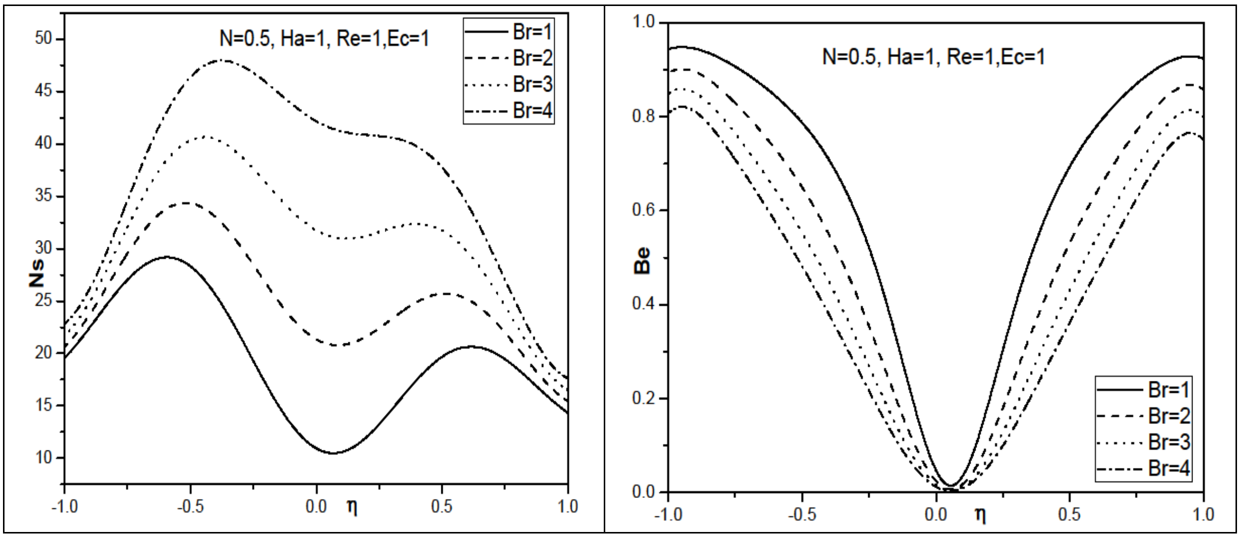

Figure 5: Effect of Brinkman number on the entropy generation and Bejan number